QUANTUM ERASER EXPERIMENT:

THE ENTITY OF COMPLEMENTARITY

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Abstract

Quantum mechanics is the backbone of modern physics. To describe how the photons' "which-path information" was erased, based on the quantum effect, we operate a series of experiments and write down the mathematical description of the phenomenon.

In general, we operate the experiment through birefringence as well as polarization, and derive the corresponding quantum states. The primary objective of the experiment is proving the quantum phenomenon by means of common tools.

Finally, we successfully erased the path information of photons, and proved the complementary principle, which involved in the Copenhagen interpretation.

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1. Devices

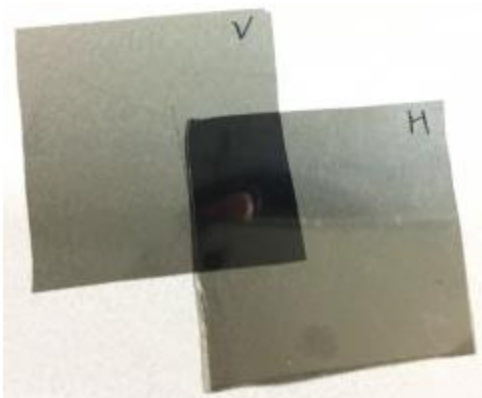
1. A green laser (650nm, <500mW)

2. A quartz crystal

3. Four polarizers (horizontal×2, vertical×2)

4. A mirror

5. A stand (assembled by Thomas)

<table>
<tr><td>The laser</td><td>The quartz crystal</td><td>The polarizer</td></tr>
<tr><td>
</td><td></td><td>
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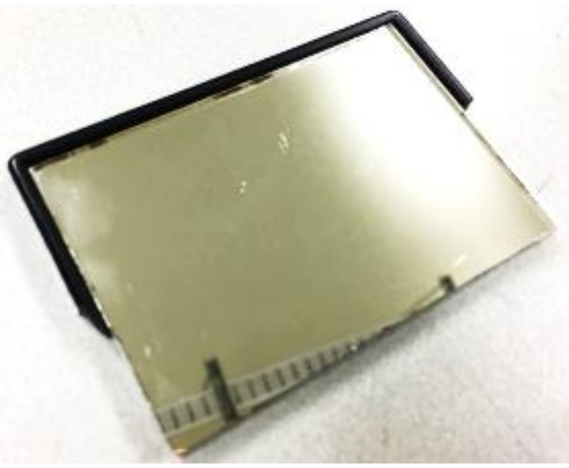
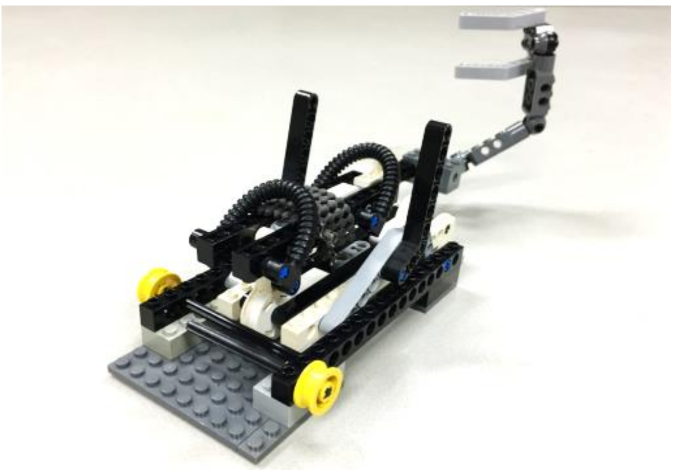
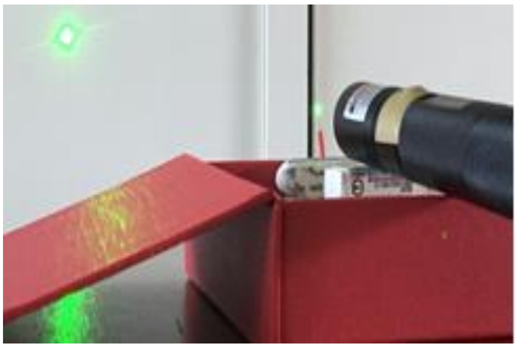
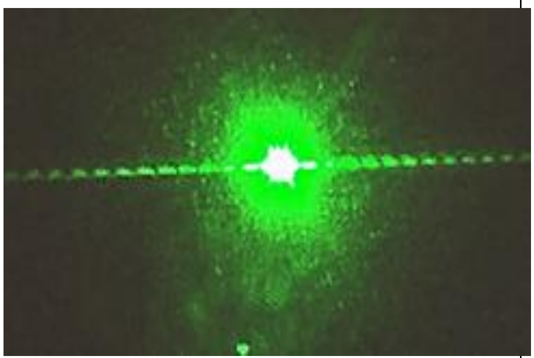
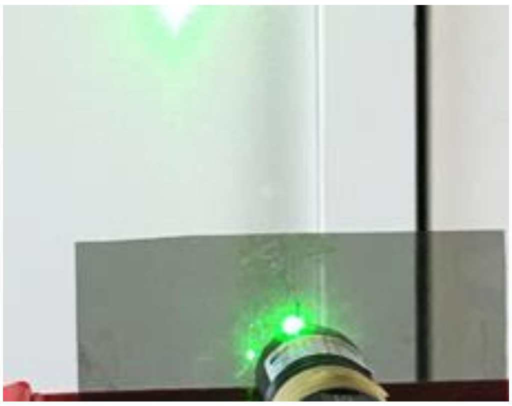
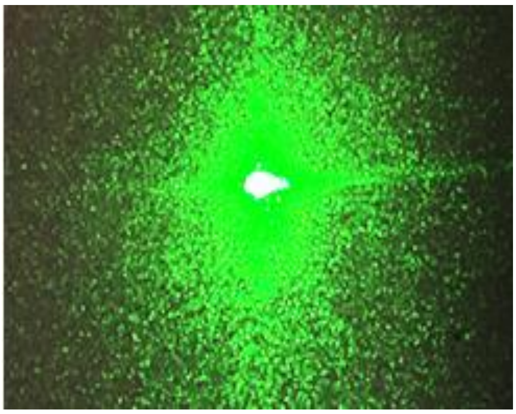
<table>
<tr><td>The mirror</td><td>The stand</td></tr>
<tr><td></td><td></td></tr>
</table>

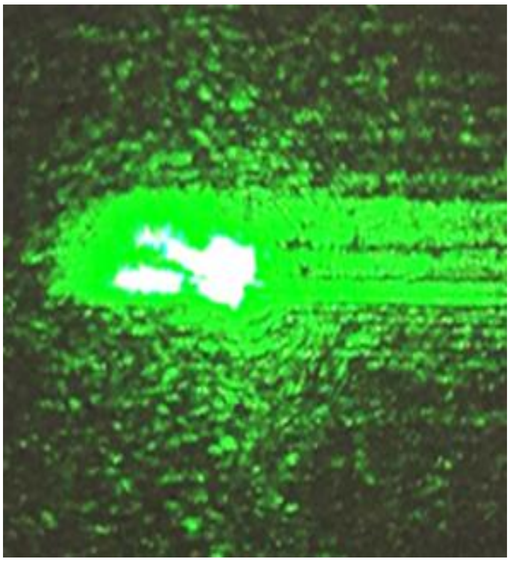
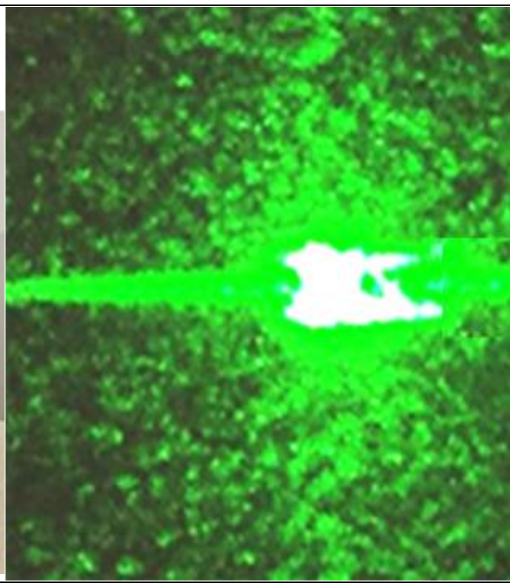
Tab 1 Our experimental devices

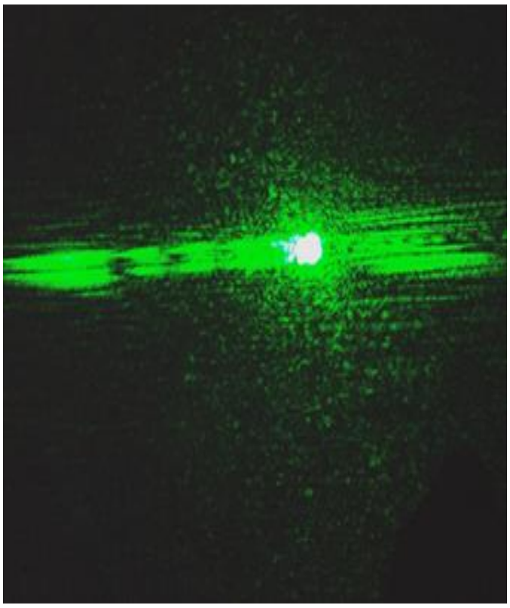
2. Experimental Process

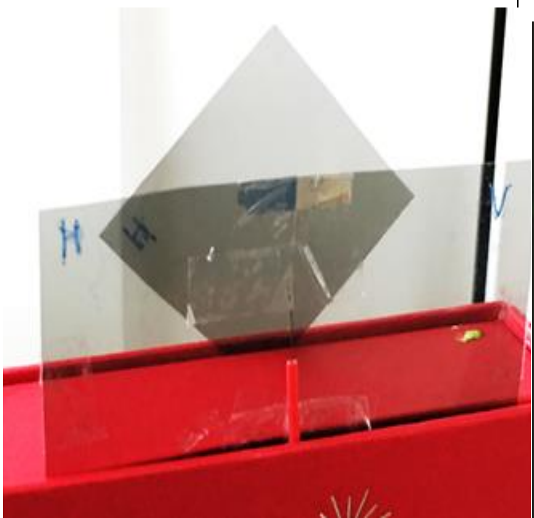
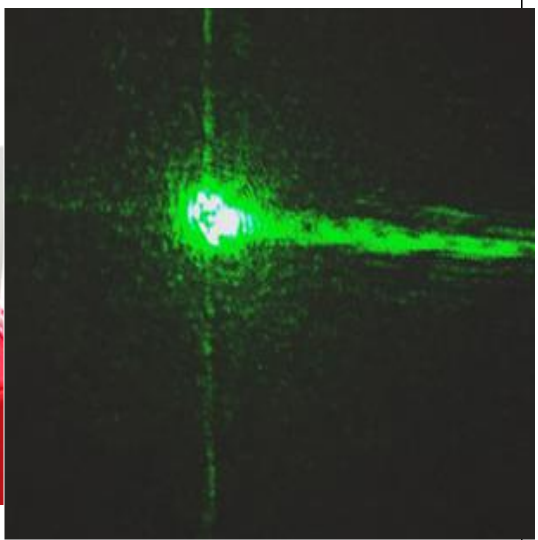
◎ **Preliminary experiment**:

(Our first experimental model)

#	*Setting*	*Phenomenon*
1st step: repeat double-slit experiment		
Introduction	We replace the double-slit with a wire, let laser go through the wire, and then send out to the screen.	The interference pattern appeared, now we cannot distinguish what path the photons went through.
Photo		
Entity	Wave	
2nd step: mark the path of photons		
Introduction	Fix the laser light source. Then, juxtapose the horizontal (H) polarizer with vertical (V) polarizer, and paste the wire in the middle of two polarizers. It is "path marker".	There's no interference pattern because the photons have been polarized, namely, we have known the path information.
Photo		
Entity	Particle	

3rd step: choose the left-photons		
Introduction	Turn the third polarizer to H direction, and place it between the path marker with the screen.	The right-photons are blocked by V polarizer and the third H polarizer. Thus, the photons in the middle of screen tend to left-direction.
Photo		
Entity	Particle	
4th step: choose the right-photons		
Introduction	Turn the third polarizer to V direction, and place it between the path marker with the screen.	The left-photons are blocked by H polarizer and the third V polarizer. Thus, the photons in the middle of screen tend to right-direction.
Photo		
Entity	Particle	

5th step: erase the path information		
Introduction	Turn V polarizer with clockwise 45°	The interference pattern appears but not obvious. Because polarizer eraser the path information of photons, the left- and right-photons can arrive the screen with probability 1/2. Now, the photon seems that pass both paths in the meanwhile and interfere with itself.
Photo		
Entity	Wave	

6th step: erase the path information with opposite direction		
Introduction	Turn V polarizer with counterclockwise 45°	Same as the former step, we can't distinguish which path the photons have passed.
Photo		
Entity	Wave	

◎ **Formal experiment**:

To make our result more accurate, we reproduce the devices and analyze the phenomenon via optics. First, let the laser emit to the uniaxial crystals which result in birefringence, and we measure by polarizers. If we know the direction of polarization, it means we gain the path information, which wouldn't appear interference pattern theoretically. Oppositely, if we don't know the direction of polarization, it means the path information was erased, which would appear interference pattern. The former represent particle entity of photons, and the latter represent wave entity of photons. Both of them are the evidence of complementary principle.

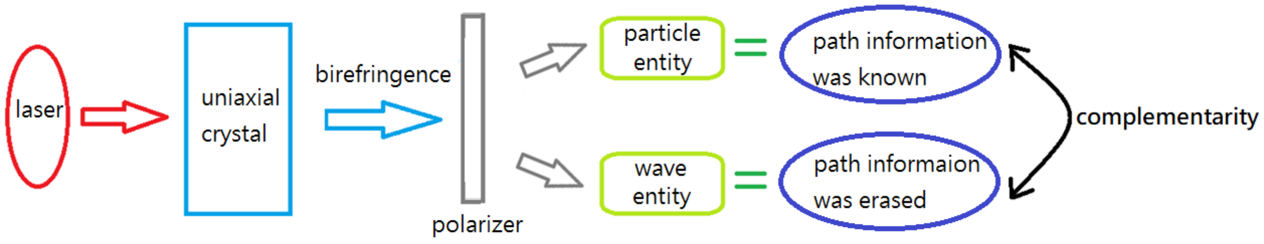
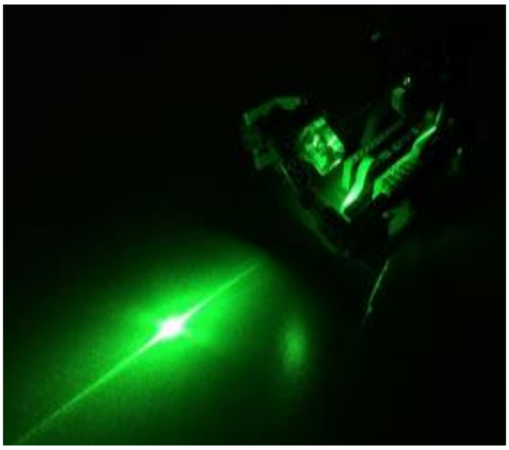
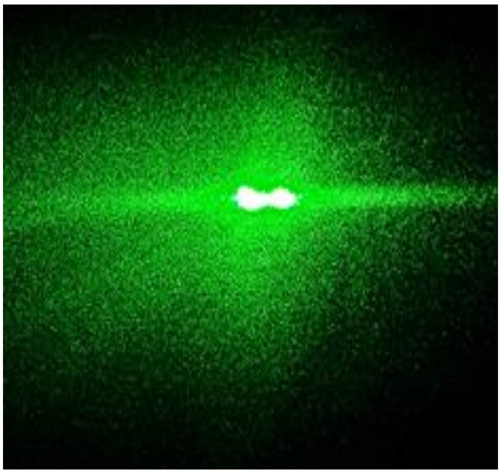
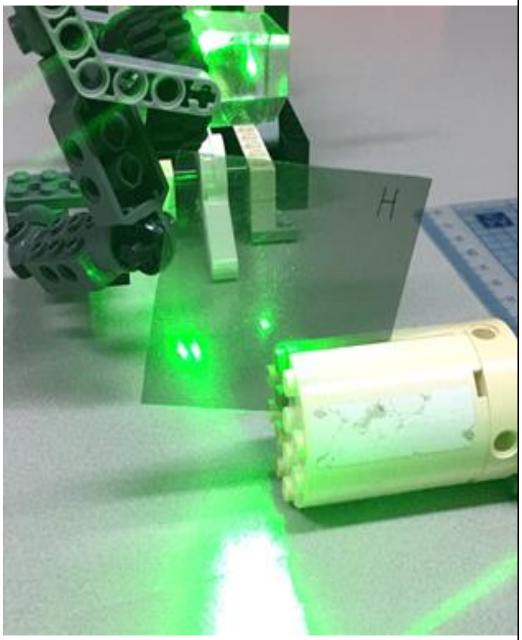
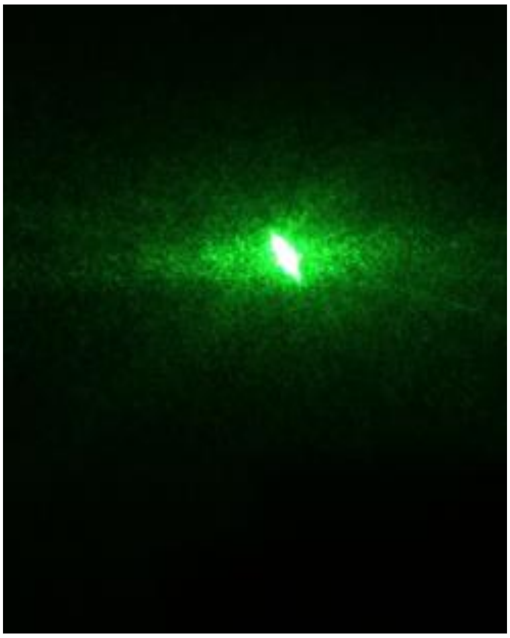
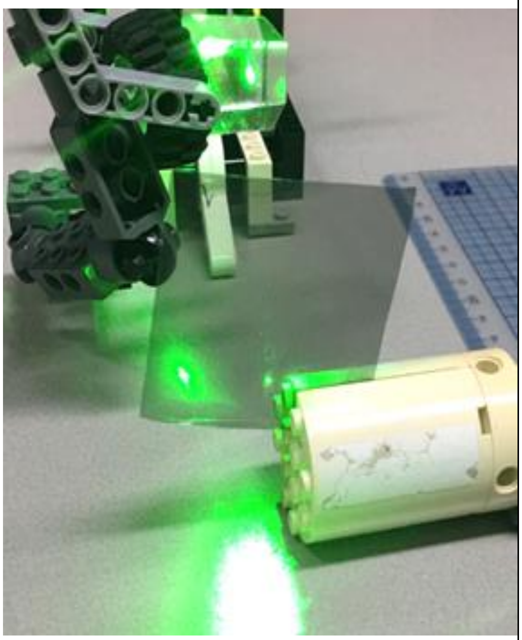
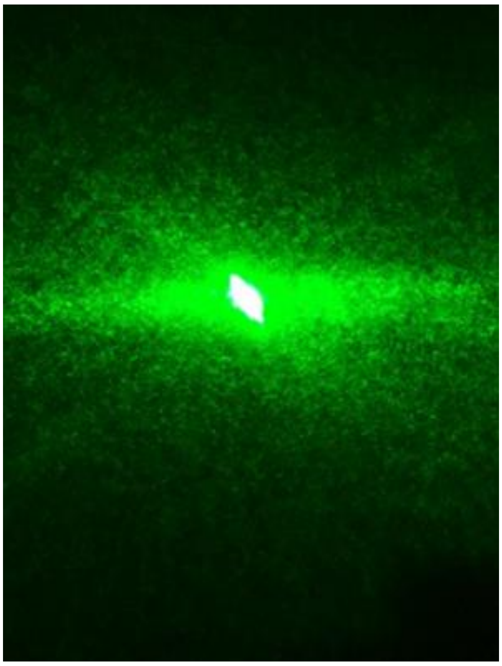


Fig 1 The diagram of our experimental process.

(1) Measure the path information (**Exp 1**)

#	*Setting*	*Phenomenon*
1st step: find the birefringent points		
Introduction	Place laser source and uniaxial crystal (quartz) on the stand, and find the birefringent points after laser passed through the crystal.	There is a pair of birefringent points appears on the screen. There is phase shift exists between two points.
Photo		

<table>
<tr><td colspan="3">2nd step: measure the direction of right-point</td></tr>
<tr><td>Introduction</td><td>Place H polarizer in the midst of crystal and birefringent points. Then, rotate the polarizer and look for the angle which the right-point is brightest.</td><td>The right-point tend to brightest whereas the left-point vanish when the included angle of polarizer and table is approximately +10°.</td></tr>
<tr><td>Photo</td><td>
</td><td></td></tr>
<tr><td colspan="3">3rd step: measure the direction of left-point</td></tr>
<tr><td>Introduction</td><td>Place V polarizer in the midst of crystal and birefringent points. Then, rotate the polarizer and look for the angle which the left-point is brightest.</td><td>The left-point tend to brightest whereas the right-point vanish when the included angle of polarizer and table is approximately +10°.</td></tr>
<tr><td>Photo</td><td></td><td></td></tr>
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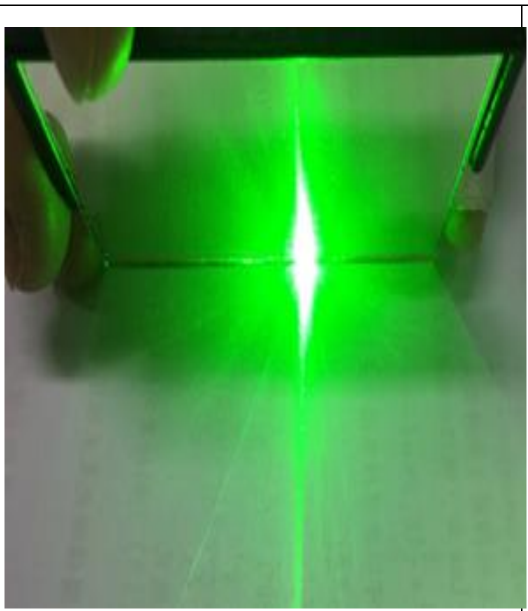
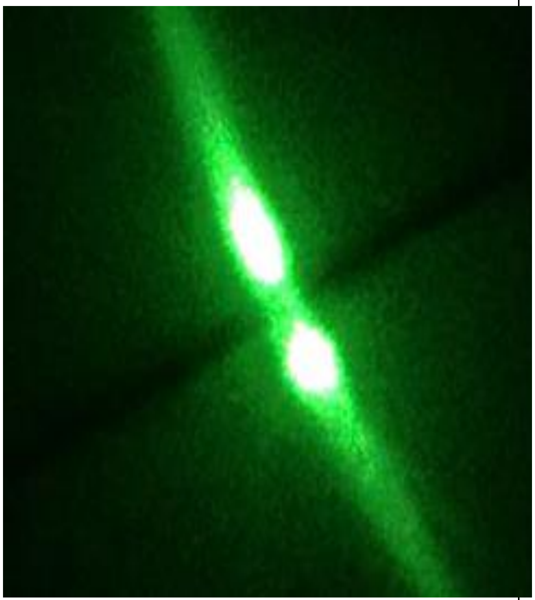
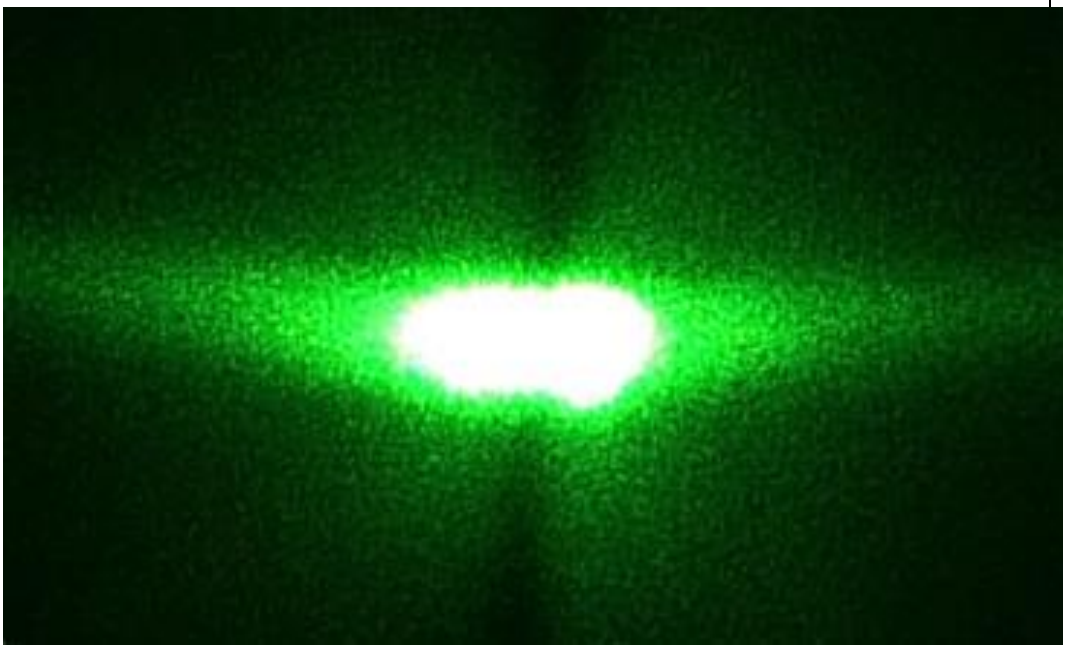
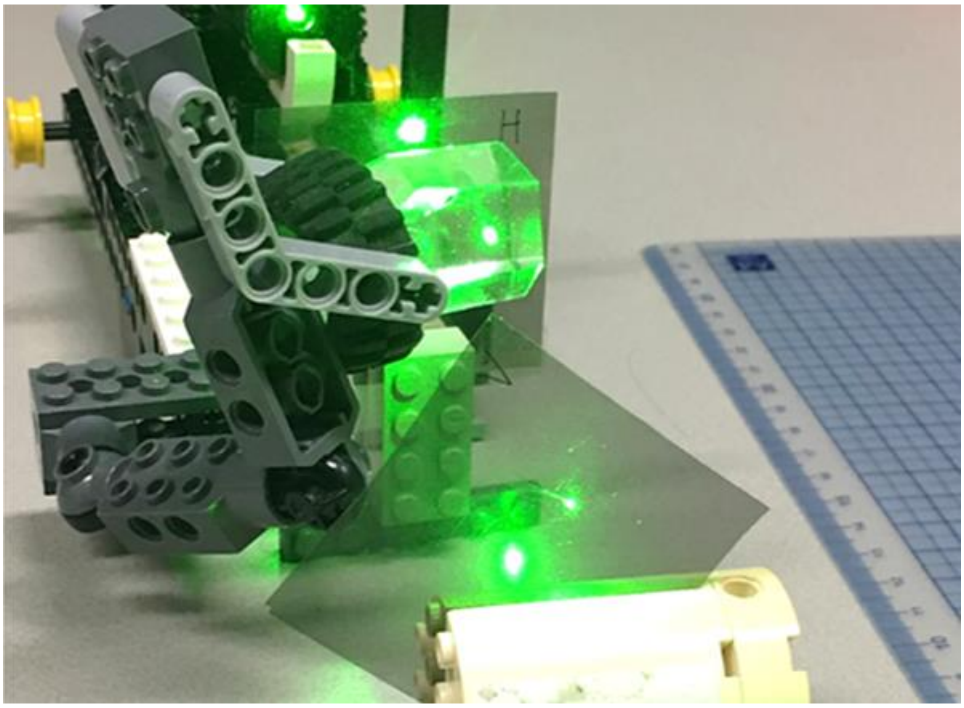
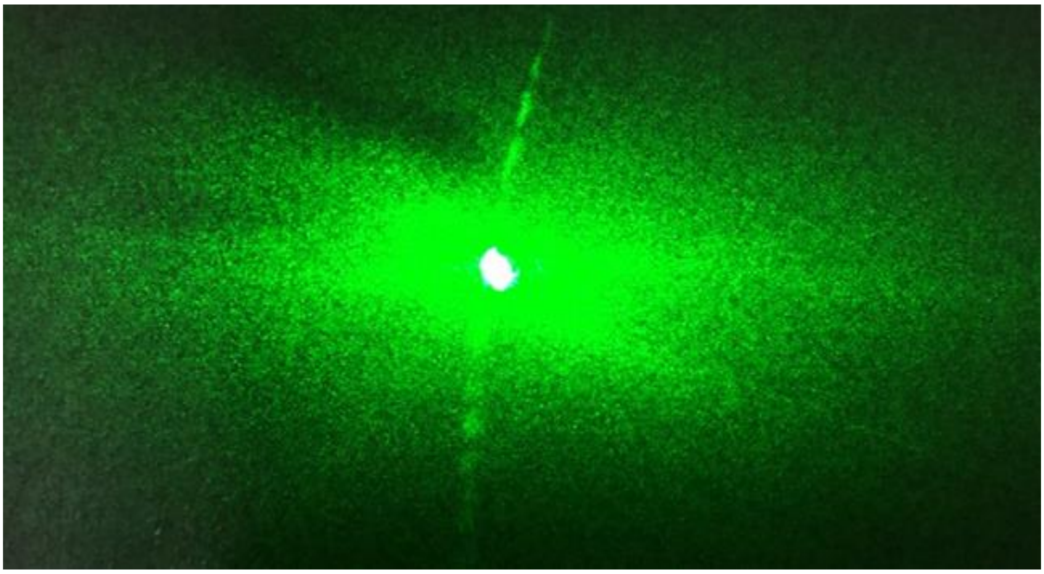
4th step: coincide two points	
Introduction	To measure whether the interference pattern appears when which-path information was known, we try coinciding two points via a mirror.
Photo	 
5th step: examine the interference pattern	
Introduction	There is no interference pattern appears, which implies path information of photons was known, namely, it represents particle entity of photons.
Photo	

(2) Erase the path information (**Exp 2**)

#	*Setting & phenomenon*
	1st step: let two beams which has phase shift become parallel
Introduction	Continue the first step of Exp 1. After found the birefringent points, we place H polarizer between laser source and crystal, and place 45° V polarizer between crystal and screen. Because two polarized beams are parallel and have phase shift, there are a pair of birefringent points appears on the screen (table).
Photo	
	2nd step: examine the interference pattern
Introduction	The interference pattern appears on the screen, which implies path information of photons was erased, namely, it represents wave entity of photons.
Photo	

3. Experimental Result

(1) Path information of photons

I. Birefringence

When laser beam passing through the uniaxial crystal, it would be divided into ordinary ray and extraordinary ray, and the latter wouldn't obey refraction law. Note we skip the phase shift during the experiment. The uniaxial crystal is similar to polarizer, namely, it can divide the incident beam into two linear polarized beams which is perpendicular to each other. Thus, the directions of two points on the screen of exp 1 are also perpendicular.

II. Microscopic path information

To compute easily, we approximate the angle of exp 1 (<10°) to 0°. The result is same because the directions of two points are orthogonal. In quantum optics, the polarizations are usually described by Jones calculus, namely, to a linear polarization which has the angle θ with x-axis, the vector representation can be written as:

$$|\theta\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

(1)

Thus, when a beam passing through the crystal in exp 1, it was divided into the H-polarized beam as well as the V-polarized beam (Fig 2). The V-polarizer is same as the 90° H-polarizer. As a consequence, the H- and the V-linear polarization can be represented as:

$$|H\rangle = \begin{pmatrix} \cos 0° \\ \sin 0° \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|V\rangle = \begin{pmatrix} \cos 90° \\ \sin 90° \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(2)

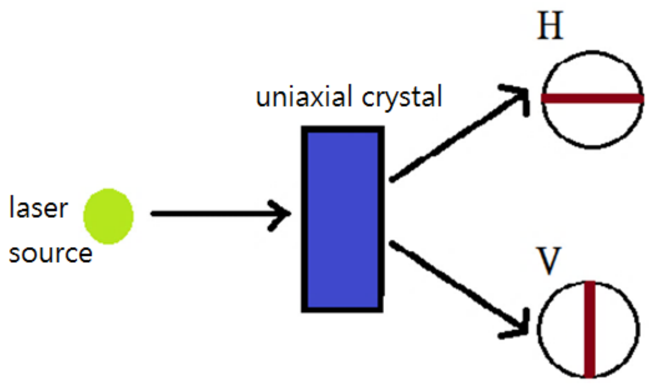


Fig 2 The diagram of exp 1: the linear polarized beams are divided via birefringence.

As for exp 2, we place a H-polarizer in front of the laser source and the crystal, and the beam was H-polarized. Then, the beam divided into H- and V-polarized beam due to birefringence, which is orthogonal to each other. Finally, two beams pass through the V-polarizer between the crystal and the screen. We rotate the V-polarizer to 45°, and two polarized beams appear parallel. In fact, the interference pattern will appear on the screen due to the phase shift of two parallel beams (Fig 3). On the whole, the interference may appear based on coherence and phase shift.

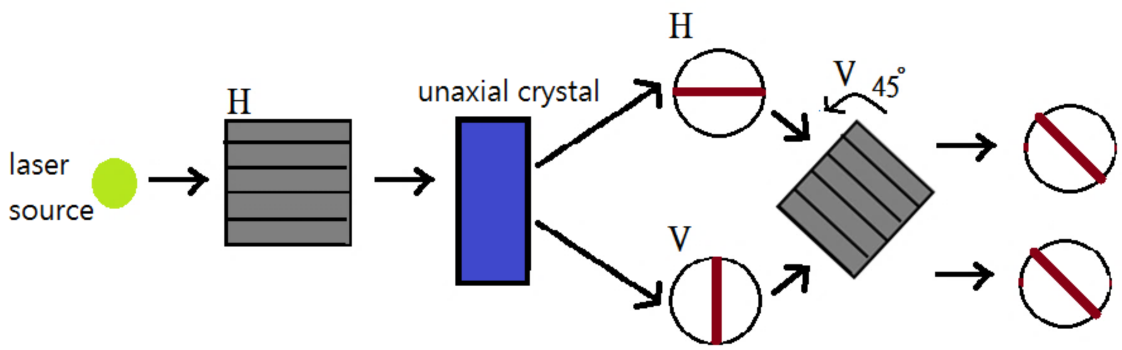


Fig 3 The diagram of exp 2: the interference appears when two parallel polarized beams with phase shift arrive the screen after passed through the polarizers.

Thus, the parallel beams after twice polarization in exp 2 have Jones vectors as below:

$$|H'\rangle = \begin{pmatrix} \cos(-45°) \\ \sin(-45°) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$|V'\rangle = \binom{\cos 45°}{\sin 45°} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

(3)

We have known the possible which-path information of photons via the processes above.

(2) Description of quantum states and probability amplitude

I. Superposition principle

To make sure whether path information was erased, we have to discuss the probability of particles' appearance. In quantum mechanics, the most typical way is writing as the form of superposition (quantum state). According to exp 1, the superposition of photons which passed through H-polarizer is:

$$|\psi_{H_1}\rangle = \cos 0°|H\rangle + \sin 0°\,|V\rangle = |H\rangle$$

(4)

It's obvious that all photons passed through H-polarizer so that the right-point is brightest whereas the left-point disappears.

Similarly, the superposition of photons which passed through V-polarizer (namely, 90°rotation of H-polarizer) can be written as:

$$|\psi_{V_1}\rangle = \cos 90°|H\rangle + \sin 90°\,|V\rangle = |V\rangle$$

(5)

Apparently, all photons passed through V-polarizer so that the left-point is brightest whereas the right-point disappears, and the polarization of left-point is V-direction.

As for exp 2, the superposition of photons which passed through H-polarizer is

$$|\psi_{H_2}\rangle = \cos(-45°)|H'\rangle + \sin(-45°)$$

(6)

And the superposition of photons which passed through V-polarizer is

$$|\psi_{V_2}\rangle = \cos 45°|H'\rangle + \sin 45°\,|V'\rangle \tag{7}$$

As a result, we can discuss the probability amplitude of photons.

2. Probability amplitude

In quantum mechanics, quantum states can be written as the superposed form of eigenvalue as well as eigenvector, namely,

$$|\psi\rangle = c_1|\lambda_1\rangle + c_2|\lambda_2\rangle + \cdots = \sum_{i=1}^{n} c_n|\lambda_n\rangle \tag{8}$$

with the eigenvalue $c_n = \langle\lambda_n|\psi\rangle$. Eigenvalue c_n is a complex coefficient which corresponding to the probability amplitude of the quantum state. Note the probability density is

$$\langle\psi|\psi\rangle = \sum_{i=1}^{n}|\psi_n|^2 = \sum_{i=1}^{n}|c_n|^2 = 1 \tag{9}$$

as a consequence of Born rule. On the other hand, according to normalizable condition, the probability is equal to probability density times the volume of space and it must equal to 1 when we solving the integral over 1-D space, namely,

$$P = \int |\psi|^2 dx = 1 \tag{10}$$

The result is important in quantum mechanics because it implies we must find a particle in a space due to the probability is equal to 1.

In exp 1, we can see the photons entirely passed through the certain direction's polarizer from (4) and (5), and the probability amplitude of them is equal to 1. Similarly, the superposed states of (6) and (7) can be written as:

$$|\psi_{H_2}\rangle = \frac{1}{\sqrt{2}}|H'\rangle - \frac{1}{\sqrt{2}}|V'\rangle$$

$$|\psi_{V_2}\rangle = \frac{1}{\sqrt{2}}|H'\rangle + \frac{1}{\sqrt{2}}|V'\rangle$$

(11)

First, according to Born rule,

$$\langle\psi_{H_2}|\psi_{H_2}\rangle = \left|\frac{1}{\sqrt{2}}\right|^2 + \left|-\frac{1}{\sqrt{2}}\right|^2 = 1$$

$$\langle\psi_{V_2}|\psi_{V_2}\rangle = \left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = 1$$

(12)

which is normalized. Then, combined with (11), we can obtain the quantum states as the following:

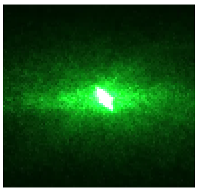
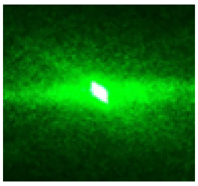
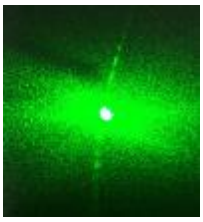
$$|\psi_{H_2}\rangle = \frac{1}{\sqrt{2}}(|H'\rangle - |V'\rangle)$$

$$|\psi_{V_2}\rangle = \frac{1}{\sqrt{2}}(|H'\rangle + |V'\rangle)$$

(13)

We can see the probability of photons in exp 2 is $\left|1/\sqrt{2}\right|^2 = 1/2$ when they passing through the second 45° V-polarizer. We can't know where the photons pass as exp 1, namely, the path information is erased. Thus, from classical viewpoint, interference comes from phase shift; from quantum viewpoint, it results from the uncertainty of observers.

Tab 2 The result of our experiment.

Polarized direction between laser source and crystal	*Polarized direction between crystal and screen*	*Probability*	*Pattern*
Before placing polarizers			
—	—	100%	
Exp 1			
—	H-direction (0°H)	Left: 0% Right: 100%	
—	V-direction (90°H)	Left: 100% Right: 0%	
Exp 2			
H-direction	45°V-direction	Left: 50% Right: 50%	

4. Discussion

(1) The real situation of polarization

In fact, we have skipped the extreme small phase shift during experiment. Generally, the laser beam would be divided into two beams of elliptical polarization after passing through birefringent crystal due to the phase shift, which can be written as:

$$\delta = \frac{2\pi\Delta}{\lambda} \tag{14}$$

with optical path difference

$$\Delta = |n_o - n_e|d \tag{15}$$

where n_o and n_e is the refractive index of ordinary ray and extraordinary ray respectively, and d is the thickness of crystal. Because the phase shift is so small that we can skip in order to easily describe the quantum states.

(2) The meaning of quantum states and probability amplitude

Taking uncertainty principle as an example, we can’t measure the accurate position and momentum of a particle at the same time. In general, the position or momentum of a particle are described as the “wave function” in quantum mechanics, which is also normalized:

$$\int |\Psi(x,t)|^2 dx = \int |\Psi(p,t)|^2 dp = 1 \tag{16}$$

The wave function is a complex function which is essential in quantum mechanics. As to the evolution of time-dependent wave function, it’s usually described by Schrödinger equation.

(3) Application of quantum eraser experiment

The generalization of quantum eraser experiment is quantum entanglement, which can be applied to quantum computer as well as quantum teleportation. On the other hand, we can also describe the “many-worlds interpretation” via the concept of our experiment. The concept is stem from Schrödinger's Cat, which implies the cat would be “both live and die” before we open the closed box. The form of superposition can be written as:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|Alive\rangle + |Dead\rangle) \tag{17}$$

The formula is similar to the quantum state of photons in exp 2. According to the many-worlds interpretation, the wave function collapses to only one state after we observe, and the other state exists in the parallel universe.

(4) The complementarity in quantum mechanics

“Complementarity” means we cannot simultaneously observe the particle-entity and the wave-entity. Such as a coin, we cannot see the head and tail at the same moment. Further, based on uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2} \tag{18}$$

When we measure the position x of a particle, we cannot measure the momentum p of a particle, and vice versa, which means the complementarity of position and momentum of a particle.

5. Conclusion

We have exhibited a few quantum phenomena of Copenhagen interpretation:

1. **Born rule**: In both exp 1 and exp 2, we derived the probability which photons passed through the certain polarizer, and all result of probability density is normalized.

2. **Correspondence principle**: Based on this principle, the quantum effect in microscope approximates to the classical phenomenon in macroscopic scale. In our experiment, we understand that interference is the result of coherence as well as phase shift. While in quantum mechanics, interference comes from the superposition of probability amplitudes. Therefore, we also exhibit correspondence principle during the experiment.

3. **Complementarity**: In the process of experiment, we not only exhibited the complementarity of particle and wave, but manifested the complementarity of which-path information and interference pattern. It's noteworthy that we cannot manifest wave-particle duality via single experiment, namely, we have to prove both entities through at least two different experiments. The fact implies the experimental result opt to the way of observation. In other words, both entities are random before observing; once we observe, the wave function would collapse to only one result (particle entity). Philosophically, the "reality" didn't exist before being observed.

Thus, we not only "erased" the path information of photons, but also understood the most mysterious fact — observation decides the experimental result.

6. Reference

- J. J. Sakurai & Jim J. Napolitano.(2010). *Modern Quantum Mechanics*. US: Pearson
- Paul Dirac.(1999). *The Principles of Quantum Mechanics*. UK: Oxford University Press
- Rachel Hillmer & Paul Kwiat.(2007). *A Do-It-Yourself Quantum Eraser.* Scientific American